Synopsis

This thesis presents work done on topological phases, Majorana modes, dynamics and transport in various system like the Kitaev models in one and two dimensions and systems with junctions of a p-wave superconductor and normal metal leads in one dimension. The systems we have studied are a one-dimensional spin-1/2 model placed in a transverse magnetic field [1,2], a lattice model of spinless electrons with p-wave superconductivity (Kitaev chain) [3,4], the Kitaev model of spin- 1/2's placed on the sites of a honeycomb lattice [5], and continuum and lattice models of one-dimensional systems with junctions of a p-wave superconductor and normal metal leads [6].

In the first chapter, we introduce a number of concepts which are used in the rest of the thesis, such as quantum phase transitions, quantum quenching, quantum fidelity, Majorana modes, Floquet theory, Floquet scattering theory, bosonization and the renormalization of an isolated impurity.

In the second chapter, we study the quenching dynamics of a many-body system in one dimension described by a Hamiltonian in which a parameter varies periodically in space. Specifically, we consider a spin-1/2 chain with equal xx and yy couplings which is subject to a periodically varying magnetic field in the $\hat{z}$ direction or, equivalently, a tight-binding model of spinless fermions with a periodic on-site chemical potential, having period $2q$, where q is a positive integer. For a linear quench of the strength of the magnetic field (or chemical potential) at a rate $1/\tau$ across a quantum critical point, we find that the density of defects thereby produced scales as $1/\tau^{q/(q+1)}$, deviating from the $1/\sqrt{\tau}$ scaling that is ubiquitous to a range of systems. We analyze this behavior by mapping the low-energy physics of the system to a set of fermionic two-level systems labeled by the lattice momentum k undergoing a non-linear quench and by performing numerical simulations. We also show that if the magnetic field is a superposition of different periods, the power law depends only on the smallest period for very large values of τ, although it may exhibit a cross-over at intermediate values of τ. Finally, for the case where a zz coupling is also present in the spin chain, or equivalently, where interactions are present in the fermionic system, we argue that the power associated with the scaling law depends on a combination of q and the interaction strength.

In the third chapter, we discuss fidelity susceptibility for one-dimensional models. Recently it has been shown that the fidelity of the ground state of a quantum many-body system can be used to detect its quantum critical points (QCPs). If g denotes the parameter in the Hamiltonian with respect to which the fidelity is computed, we find that for one-dimensional models with large but finite size, the fidelity susceptibility χ_F can detect a QCP provided that the correlation length exponent satisfies $\nu < 2$.

We then show that χ_F can be used to locate a QCP even if $\nu \geq 2$ if we introduce boundary conditions labeled by a twist angle $N\theta$, where N is the system size. If the QCP lies at $g = 0$, we find that if N is kept constant, χ_F has a scaling form given by $\chi_F \sim \theta^{-2/\nu} f(g/\theta^{1/\nu})$ if $\theta \ll 2\pi/N$. We illustrate this both in a tight-binding model of fermions with a spatially varying chemical potential with amplitude h and period $2q$ in which $\nu = q$, and in a XY spin-1/2 chain in which $\nu = 2$. Finally we show that when q is very large, the model has two additional QCPs at $h = \pm 2$ which cannot be detected by studying the energy spectrum but are clearly detected by χ_F. The peak value and width of χ_F seem to scale as non-trivial powers of q at these QCPs. We argue that these QCPs mark a transition between extended and localized states at the Fermi energy.

In the fourth chapter, we present a comprehensive study of two extensions of the Kitaev model of a one-dimensional spinless p-wave superconductor: those involving longer range hopping and superconducting pairing. We begin with a review of the Kitaev model. In order to characterize the topological phases exhibited by this system, we introduce bulk topological invariants as well as those derived from a consideration of the boundary modes. In time-reversal symmetric systems, we find that the longer range hopping leads to topological phases characterized by multiple Majorana modes. In particular, we investigate a spin model, which respects a duality and maps to a fermionic model with multiple Majorana modes; we highlight the connection between these topological phases and the broken symmetry phases in the original spin model. In the presence of time-reversal symmetry breaking terms, we show that the topological phase diagram is characterized by an extended gapless regime.

In the fifth chapter, we show how Majorana end modes can be generated in a one-dimensional system by varying some of the parameters in the Hamiltonian periodically in time. The specific model we consider is a chain containing spinless electrons with a nearest-neighbor hopping amplitude, a p-wave superconducting term and a chemical potential; this is equivalent to a spin-1/2 chain with anisotropic XY couplings between nearest neighbors and a magnetic field applied in the $\hat{z}$ direction. We show that varying the chemical potential (or magnetic field) periodically in time can produce Majorana modes at the ends of a long chain. We discuss two kinds of periodic driving, periodic δ-function kicks and a simple harmonic variation with time. We discuss some distinctive features of the end modes such as the inverse participation ratio of their wave functions and their Floquet eigenvalues which are always equal to ± 1 for time-reversal symmetric systems. For the case of periodic δ-function kicks, we use the effective Hamiltonian of a system with periodic boundary conditions to define two topological invariants. The first invariant is a well-known winding number while the second invariant is new. The second invariant is more powerful in that it always correctly predicts the numbers of end modes with Floquet eigenvalues equal to $+1$ and -1, while the first invariant does not. We find that the number of end modes can become very large as the driving frequency decreases. We show that periodic δ-function kicks in the hopping and superconducting terms can also produce end modes. Finally, we study the effect of electron-phonon interactions (which are relevant at finite temperatures) and a random noise in the chemical potential on the Majorana modes. Next, we study the Majorana modes, both equilibrium and Floquet, which can appear at the edges of the Kitaev model on the honeycomb lattice. We first present the analytical solutions known for the equilibrium Majorana edge modes for both zigzag and armchair edges of a semi-infinite Kitaev model and chart the parameter regimes in which they appear. We then examine how edge modes can be generated if one of the Kitaev couplings is varied in time as periodic δ-function kicks. We derive a general condition for the appearance and disappearance of the Floquet edge modes as a function of the driving frequency for a generic d-dimensional integrable system. We confirm this general condition for the Kitaev model with a finite width by mapping it to a one-dimensional model. Our numerical and analytical study of this problem shows that Floquet Majorana modes can appear on some edges in the kicked system even when the corresponding equilibrium Hamiltonian has no Majorana mode solutions on those edges. We support our analytical studies by numerics for finite sized system which show that periodic kicks can generate modes at the edges and the corners of the lattice.

In the sixth chapter, we study Majorana modes and transport in some one-dimensional systems with junctions of a p-wave superconductor and normal metal leads. For a system with a superconductor lying between two normal metal leads, it is known that there is a Majorana mode at the junction between the superconductor and each normal metal. If a strong impurity is present or the p-wave pairing amplitude changes sign at some point in the superconductor, two additional Majorana modes appear near that point. We study the effects of all these modes on the sub-gap differential conductances between the leads and the superconductor. The main effect is to shift the conductance peaks away from zero bias due to hybridization between the Majorana modes; the shift oscillates and decays exponentially as the length of the superconductor is increased. Using bosonization and the renormalization group method, we study the effect of interactions between the electrons on the Majorana modes and on the conductances. We propose experimental realizations of these systems. Next we study the normal and Cooper conductances for time-independent and time-dependent NNN and NSN system using lattice models. We study the effect of time-dependent μ on conductance peaks. We find that side bands appear in the conductance peaks.