Synopsis

One of the recent fundamental developments in theoretical high energy physics is the AdS/CFT correspondence [1, 2, 3, 4] which posits a relationship between Quantum Field Theories (QFT) in a given dimension and String Theory on a higher dimensional anti- de Sitter (AdS) spacetime. This has revolutionised our understanding of QFTs (more specifically conformal field theories (CFTs)) and string theory/gravity, and has far reaching consequences for explorations into a vast array of physical phenomena. Using the elegant formalism provided by this powerful duality, often called "holography", one can now use fundamental physical observables in QFT to better understand the nature of quantum gravity. The theoretical tools provide a translation of calculable field theoretic observables into the language of gravity thereby leading to the construction of holographic models for several interesting QFTs.

Entanglement is a fundamental physical property of all quantum systems. From models of various condensed matter systems to its application as a tool for secure and fast communication in quantum information theory [5], it serves as an intersection point between different subfields of physics [6]. From the AdS/CFT point of view quantum entanglement connects geometry with quantum information, providing a window to understand how the bulk gravity physics emerges from the holographic field theoretic viewpoint. Probing various aspects of this connection in detail will be the broad theme of this thesis.

For extended, many-body systems, the most well known measure of quantum entanglement is the "Entanglement Entropy" (EE) which is also the best understood measure within the holographic framework. In early 2006, Ryu and Takayanagi (RT) gave a simple and elegant prescription for computing this quantity using AdS/CFT duality within Einstein gravity [7, 8]. They proposed that EE for a subsystem within an extended system (QFT), is computed by the (proper) area of a static, codimension- 2, "extremal" surface inside the dual AdS spacetime. The RT proposal has passed several non-trivial consistency checks, for example strong sub-additivity, area law to name a few [9]. A remarkable aspect of the proposal is the ease with which EE can now be calculated, while it is well known that obtaining EE from first principles in QFT presents several technical challenges which have so far been surmounted only in some 2d field theories using the "replica method" [10, 11, 12].

The most intriguing aspect of the RT proposal is its striking similarity to Bekenstein-Hawking (BH) entropy which is proportional to the area of a black hole horizon, further confirming an intimate relationship between entropy and geometry [13, 14, 15, 16]. This leads to the natural question: what is the connection between EE and BH entropy? This question has been sharpened recently by Lewkowycz and Maldacena (LM) via the concept of Generalized Gravitational Entropy which extends the QFT replica trick to a replica symmetry for the dual space-time [17]. This was used to prove the RT conjecture successfully by deriving the correct extremal surface equation for two derivative gravity theories. In this thesis I have studied the generalization of LM method for higher derivative gravity theories [18, 19, 20, 21, 22, 23] describing holographic duals (of QFT's with finite number of colours) and finite 't Hooft coupling which takes the AdS/CFT correspondence beyond the usual supergravity limit. If one wants to use AdS/CFT to study real life systems then it is absolutely necessary to incorporate the finite coupling effect into the theory and hence the study of higher derivative effects becomes very important. In these two papers [21, 22] I have formulated a proof for the existence of the entropy functionals for certain higher derivative theories extending LM method. We have shown that the for a certain special class of higher derivative theories there exist well defined entropy functionals. To extend this proof for more general theories of gravity opens up a possibility of breaking replica symmetry in the bulk space-time [24].

For higher derivative gravity, black hole entropy for a large class of stationary black holes with bifurcate killing horizon is given by the well known Wald prescription [25, 26, 27] which relates the concept of the Noether charge with the black hole entropy. Iyer and Wald proposed a generalization for dynamical horizons. This throws up the question whether there is a relation between these EE functionals and the Noether charge, and whether we can derive them using the approach of Iyer and Wald. For a certain class of theories I have shown that there exists a relation between these two [28] but a more rigorous proof is needed. This somewhat firms up the area-entropy relation for arbitrary surfaces and proves the existence of holographic EE functionals for higher curvature theories thereby extending the applicability of Iyer-Wald formalism beyond the bifurcation surface.

Apart from this, it is well known that there exist several measures of quantum entanglement, each satisfying a variety of mathematical inequalities and conditions [5]. Translating these into the language of holography constrains the dual gravity theory and will lead to general statements about the consistency of the theory. In this thesis I have discussed one such measure namely Relative entropy [29], the positivity of which has led to constraints on the underlying gravity theory [30]. Also entanglement entropy is a very useful tools for probing renormalization group (RG) flow from the holographic point of view [34, 31, 32, 35, 33]. We end with exploring the concept of renormalized entanglement entropy [36, 37] and its application in probing RG flow in the context of $\mathcal{N} = 2$ gauged supergravity [38].

References

[1] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998), [arXiv:hep-th/9711200]