Abstract

Cosmic Microwave Background (CMB) is a relic from the early Universe. It was generated due to the physical processes in the early Universe during an epoch known as the recombination or decoupling epoch. The CMB has highly uniform temperature over the entire sky but with small variations in different directions. The Thomson scattering between photons and electrons during the decoupling epoch, which contain quadrupole anisotropies results in the linear polarization of the CMB. The CMB radiation along each line of sight is associated with temperature (T) and polarization. The polarization can be decomposed into Stokes parameters Q=U, or E mode (E) and B mode (B) fields. Here, Q=U fields transform as spin +_2 objects under rotation transformation while the E=B fields remain invariant. The fluctuations observed in CMB is due to the quantum fluctuations generated during the inflationary phase, which is a period of exponential expansion moments after the Big Bang in the early Universe. The statistical properties of CMB fluctuations will be similar to the primordial fluctuations for the linear evolution of fluctuations. The statistical observables are used to capture the morphological properties of the CMB fluctuations. Then the morphological properties can be studied in relation to the parameters describing the physical mechanisms of the inflationary phase. In this research work, we use the geometrical and topological observables to study the CMB polarization fields and further we also introduce a novel statistical observable known as the Tensor Minkowski Functionals (TMFs) for the analysis of CMB fields.

The models about the inflationary phase predict that the Probability Distribution Function (PDF) of primordial fluctuations are close to the Gaussian distribution with small deviation. The information about the exact form of deviation in the PDF of primordial fluctuation is encapsulated in the CMB Fields. We investigate the local type non-Gaussian features in the CMB polarization fields, which is parametrized by fNL. We use the simulations of local type non-Gaussian CMB fields, namely T, E and IP, and study the deviation in their PDF relative to the Gaussian distribution. The numerical calculations show that the non-Gaussian deviation in the PDF of E field is similar to that of the T field. While the non-Gaussian deviation corresponding to the IP field has smaller amplitude and large error bars in comparison to that of T field. This analysis was repeated using the geometrical and topological observables, namely Scalar Minkowski Functionals (SMFs) and Betti numbers of fields. These observables capture different morphological features of a given field. The results obtained using these observables are similar to those from the PDF of the Fields. Hence from the theoretical point of view, these results imply that the E field can provide an independent constraint on fNL similar to the T field. Further, the results show that when the IP field is independently used for such analysis, it cannot provide any statistically significant information. In the realistic scenario, the observational data contains instrumental systematics which will lead to the reduction in the statistical significance of the above results.

The CMB polarization is usually analyzed using the E=B fields as they are scalar fields. We investigate the theoretical aspects of using the Q=U fields as a complementary analysis of CMB polarization. We show that the variance of Q=U and its gradient Fields are invariant under rotation transformation; hence it follows that the SMFs of a Gaussian Q=U fields is invariant. However, this statement breaks down for incomplete sky. Then we studied the non-Gaussian

deviation in Q=U fields constructed from the simulations of local type non-Gaussian E field. These simulations use the same x?y coordinates along each line of sight. We found that its amplitude is about an order of magnitude lower than that of T Filed and has different shape. This finding will be useful for distinguishing different non-Gaussian signals in the observational data from future experiments. Further, we studied the effect of the presence of primordial tensor perturbation, which is parametrized by r, on the SMFs of Q=U and IP fields, and the number density of singularities in IP Filed. Here, a singularity is a point on the CMB field where IP = 0. We found that the amplitude of SMFs of these Fields are sensitive to the presence of primordial tensor perturbation and it decreases with r. We also show that the number density of singularities in IP Filed decreases with r. This finding will be useful for the searches of primordial tensor perturbation in the future experiments. The instrumental systematics in the observational data will decrease the statistical significance of the above results.

TMFs are tensor generalization of Minkowski Functionals which we introduce a new statistical observable for the analysis of CMB data. Since these are tensor quantities, they are capable of capturing more morphological properties in a given Filed than their scalar counterparts. We have developed a code, referred as TMF Code, to compute the TMFs for any general Filed on a Euclidean plane. In order to apply the TMFs, specifically W1;1 2 which is a tensor of rank 2, to CMB Fields which lies on a 2-d spherical surface, we map each point on the sphere with a point on a plane using stereo-graphic projection. The code computes W1;1 2, and then the net anisotropy (_) and net orientation (_) of the structures are estimated. We investigated the numerical error in this computation due to pixelization. We found the error in _ increases with the increasing curvature of the boundaries of the structure. The error in _ is negligible when the structures are completely unoriented with each other and it increases as the structures become more and more aligned with each other. We present the numerical calculation of the systematic variation of A and B with the threshold value for the simulated Gaussian and isotropic CMB T and E Fields. We found that the value of _ shows a characteristic variation with the threshold value while _ is at. We show that according to the standard model, _ = 0:62 for T and _ = 0:63 for E, where the values are corrected for pixelization. The value of _ is one for both the Fields, which is as expected for an isotropic Filed.

We applied W1;1 2 for the analysis of PLANCK data as an illustration of its application. The instrumental systematics and the gravitational lensing due to large scale structure affects the morphological features of the CMB Fields. We study the effect of these factors on the value of A and Busing the simulations of CMB frequency bands, namely 44GHz and 70GHz provided in PLANCK data, which contains the respective instrumental characteristics. We found that the percentage difference in A and B due to these factors are less than 2% and it significantly increases the size of their error bars. We use the CMB simulations corresponding to the frequency band 44GHz as the basis for testing the consistency of different PLANCK data sets with theoretical expectations. We estimated the deviation in A and B for the foreground cleaned CMB maps namely SMICA, COMMANDER, SEVEM and NILC corresponding to full mission, half mission 1, half mission 2, half ring 1 and half ring 2 provided in the PLANCK data. These calculations showed that B is consistent with the standard model within 2_ for all data sets, except the T map of NILC half mission 2 which has slightly higher deviation. We found the values of _ for T map of different data sets to be in excellent agreement with the standard model within 1:2_. The deviation in _ of E map of all data sets are higher than 3_ except the SMICA full mission data. Further, _ for E map corresponding to the half mission 1 of all data sets showed consistently higher deviation of 5 These results imply that the structures in the E map has an extent of alignment with each other. This alignment could be cosmological or due to instrumental systematics. Since we are comparing the PLANCK maps which are obtained by co-adding all frequency bands with that of the simulations with the instrumental characteristics of a specific frequency band, namely 44GHz, the instrumental systematics is more probable reason for the alignment measured in E map.