Abstract

The thesis touches numerical solutions and wave propagation analysis using spectral methods for different classes of *partial differential equations* (PDEs) having applications in various fields. We address the solution of both linear and nonlinear PDEs. The different problems studied in the thesis are the one dimensional (1-D) wave equation for uniform and varying cross-sectional area, the coupled one dimensional Timoshenko beam with uniform and varying cross-section, the three dimensional linear Heat equation having real solution, the two dimensional linear Schrodinger equation having oscillatory solution, one dimensional nonlinear Korteg-de-Vries equation (KdV), two dimensional nonlinear Schrodinger (NLS) equation, and finally the coupled one-dimensional (1-D), two-dimensional (2-D) and three dimensional (3-D) quasicrystals with four, five and six variables.

The major objective of the work is to bring out the versatality of the spectral methods to solution for the above listed equations. The solutions to these partial differential equations discussed in the thesis has been obtained by approximating the unknown function using *spectral functions* such as Legendre or Chebyshev polynomials as basis function in the frequency domain and also in the time domain. For the nonlinear partial differential equations, we have obtained solutions using Fourier spectral functions along the spatial direction in time domain.

Another important area that was studied in the thesis is the *wave propagation* in the frequency domain. Wave propagation is a transient behaviour resulting from short duration loading, which have high frequency content. The key factor in the wave propagation is the propagating velocity of the waves, the level of attenuation of their response and their wavelengths. It is a multi-modal phenomenon and hence the analysis becomes difficult if the problem is solved in the time domain. This is because the problem by its nature is a high frequency content problem, where the wavelengths are small and the mesh sizes should be of the order of the wavelength, which makes the problem sizes very large. We have discussed the wave propagation analysis in rods, Timoshenko beam and 1-D, 2-D and 3-D quasicrystals. We have also shown that the numerical spectral methods effectively evolve the physical behaviour of the above equations.

An initial introduction to the spectral finite element method in frequency domain is given for the solution of partial differential equations considering the 1-D wave equation. Signal wrap around problems are observed as a result of using Discrete Inverse Fourier Transform and different modelling strategies for avoiding the signal wrap around effects are discussed. One way of avoiding signal wrap around is by adding sufficient damping to the response, by approximating in

time using numerical Laplace transform, which is used throughout for the analysis of the above equations. Different numerical methods such as Galerkin Method, Petrov- Galerkin method, Method of moments and Collocation method or the Pseudo-spectral method in frequency domain are studied and compared with the exact solution. For comparison of the time domain analysis with frequency domain analysis, we seek an approximate solution in both frequency domain and time domain and then the approximate solution is compared with the exact solution. The different spectral methods are compared for the mean squared error MSE and maximum error Err_{max} in the case of the 1-D wave problem. The effect of the number of segments S, number of FFT points N_s and the order of the polynomial N are studied and compared with the exact solution. The computational time required for all the above methods and also for different grids points are also studied and compared with the exact solution.

The wave propagation analysis in 1-D mechanical waveguides constitutes one of the major areas of the research work. For this, we consider the spectral element method of solution in frequency domain using spectral functions for the numerical solution. Here, we consider the Cantilever rod with uniform and varying cross-section and the Timoshenko beam with uniform and varying cross-section. The exact solution of the Cantilever beam with uniform and varying cross-section and the Timoshenko beam with uniform cross-section is available. However, the exact solution for Timoshenko beam with varying cross-section is not available. Initially we use the Laplace spectral methods to solve these problems exactly in frequency domain. An approximate solution is also obtained for the Timoshenko beam with varying cross-section using Laplace Spectral Element Method (LSFEM).

The effect of the frequency f of the input pulse on the group speeds c_g for the Cantilever beam and the Timoshenko beam are also studied. It is shown that the group speeds are a constant for the Cantilever rod with uniform cross-section and the group speed vary with frequency f for the Timoshenko beam. It is also shown that a modulated pulse is able to extract the shear mode and the bending mode of the Timoshenko beam with uniform and varying cross-section. The shear mode and the bending modes of the Timoshenko beam with uniform cross-section are separated numerically by applying a modulated pulse as the shear force and the corresponding group speeds for varying taper parameter m are obtained numerically by varying the frequency of the input pulse.

For the Timoshenko beam of varying cross-section, we obtain the wavenumber and group speeds numerically using Laplace Spectral Element Method using spectral function approximation in the spatial domain. An approximate expression for calculating group speeds corresponding to the shear mode and the bending mode of the Timoshenko beam of varying cross-section. The expression for cut-off frequency is also obtained. This is validated using the frequency domain method.

We also consider spectral method of solution for higher dimensional problems also having real and oscillatory solutions having a complex wave field. The solution of the 3-D transient heat equation and the 2-D time dependent Schrodinger equation is approached using spectral methods along the spatial direction in the frequency domain. The two problems have been selected for study so as to compare the Laplace method of solutions for real and oscillatory solutions.

Another area of the research work is the 1-D and 2-D nonlinear partial differential equations with initial condition. For this we have considered the 1-D Korteg-de-Vries (KdV) and the 2-D nonlinear Schroedinger (NLS) equation with specified initial conditions. The equations are first solved using Fourier spectral approximation in the spatial domain and is compared with the results of approximation in the spatial domain using orthogonal polynomial functions such as Legendre or Chebyshev polynomials as their basis functions.

Another major area of the research contribution is the wavepropagation in quasicrystals. We obtain the wave propagation of the phonon and the phason displacement modes in 1-D, 2-D and 3-D quasicrystals. An introduction to quasicrystals and the elasto-hydrodynamic equations of 1-D, 2-D and 3-D quasicrystals are presented. We have then obtained the wave propagation in different quasicrystals. The spectral analysis (in frequency domain) of the elasto-hydrodynamic equations are performed to study the propagation of different waves in quasicrystals. The wavenumber (k_n) and the group speeds (c_g) of the phonon and the phason displacement modes of the 1-D hexagonal, 2-D decagonal quasicrystal and the 3-D icosahedral quasicrystals are computed using spectral method in the frequency domain. This will help in analysing the phonon and the phason displacement modes of various quasicrystals. For the computation of the wavenumber and group speeds, we have used Fourier transform approximation of the phonon and the phason displacement modes.

Finally, we consider wave propagation in different quasicrystals on a beam type structure using 2-D spectral element formulation. An Aluminium Cantilever beam is reinforced with a layer of quasicrystal under different orientation and the wave propagation characteristics of the hybrid structure is studied. The analysis is performed using frequency domain spectral finite element formulation. The analysis considers different 2-D decagonal and 3-D icosahedral quasicrystals. The study includes the propagation of axial and transverse wave responses in these quasicrystals. For all the combinations of quasicrystal Aluminium beam combination, there is substantial suppression of responses both for the axial and the bending responses. Unsymmetrical configuration produces substantial nondominant phonon modes which propagate dispersively. We have shown that for a symmetric bi-morph configuration, the response is reduced significantly.

The whole thesis comprises of 8 chapters, where we have concentrated on spectral methods of solution of different classes of PDEs and then the wave propagation analysis, which is mainly done on 1-D mechanical waveguides and quasicrystals.