

ABSTRACT

This thesis concerns dilation theory, analytic models, joint invariant subspaces, reproducing kernel Hilbert spaces and multipliers associated to commuting tuples of bounded linear operators on Hilbert spaces. The main contribution of this thesis is twofold: dilation and analytic model theory for n -tuples of (1) commuting contractions (in the setting of the unit polydisc), and (2) commuting row contractions (in the setting of the unit ball). On n -tuples of commuting contractions: We study analytic models of operators with some positivity assumptions and quotient modules of function Hilbert spaces over polydisc. We prove that for an m -hypercontraction $T \in \mathcal{C}(\mathcal{H})$ on a Hilbert space \mathcal{H} , there exist Hilbert spaces E and E_α , and a partially isometric multiplier $\mu \in H^\infty(E, D)$, $A \in m(E_\alpha) \subset \mathcal{C}(E_\alpha)$ such that $\mathcal{H} \cong \mathcal{A} \oplus Q_\mu \mathcal{A} \oplus A \oplus m(E_\alpha) \oplus \mu H^\infty(E, D)$, and $T \cong \mathcal{A} \oplus P Q_\mu M_z \oplus Q_\mu$, where $A \in m(E_\alpha)$ is the E_α -valued weighted Bergman space and $H^\infty(E, D)$ is the E -valued Hardy space over the unit disc D .

We then proceed to study and develop analytic models for doubly commuting n -tuples of operators and investigate their applications to joint shift co-invariant subspaces of reproducing kernel Hilbert spaces over polydisc. In particular, we completely analyze doubly commuting quotient modules of a large class of reproducing kernel Hilbert modules, in the sense of Arazy and Engliš, over the unit polydisc D^n . On commuting row contractions: We study wandering subspaces for commuting tuples of bounded operators on Hilbert spaces. We prove that for a large class of analytic functional Hilbert spaces HK on the unit ball in C^n , wandering subspaces for restrictions of the multiplication tuple $M_z \in (M_{z_1}, \dots, M_{z_n})$ can be described in terms of suitable HK -inner functions. We prove that, HK -inner functions are contractive multipliers and deduce a result on the multiplier norm of quasi-homogenous polynomials as an application. Along the way we prove a refinement of a result of Arveson on the uniqueness of minimal dilations of pure row contractions.