

# Abstract

This thesis is a study of the sound ray dynamics in range-dependent ocean environments. The analysis takes place in the mathematical framework of the theory of non-linear dynamics of conservative or Hamiltonian systems. In range-dependent ocean environments, the acoustic ray equations which have Hamiltonian form, are non-integrable. Consequently, at least some of the ray trajectories are expected to exhibit chaotic behaviour. Chaotic rays are characterized by extreme sensitivity to initial conditions due to an exponential divergence of neighbouring rays. The exponential divergence imposes a limit on the range of predictability of chaotic rays, known as predictability horizon. In this thesis, numerical analysis of chaotic ray trajectories has been carried out for both shallow and deep ocean models. In the shallow ocean model, range-dependence is due to the surface which has been modeled both as a sinusoid and as a superposition of random sinusoids. In the deep ocean model with flat reflecting surface and bottom, a canonical sound-speed profile is perturbed by a random field of internal waves. Existence of chaotic rays has been shown with the help of Poincare maps. The chaotic behaviour is quantified by computing the Lyapunov exponent, and numerical values of the predictability horizon are obtained for the different ocean models. It is found that the predictability horizon is only a few hundred meters to a few kilometers in the shallow ocean and a few tens of kilometers in the deep ocean even for very small perturbation amplitudes. The practical importance of chaotic ray trajectories—a limitation on one's ability to make deterministic predictions using ray theory—is emphasized.