Abstract

This thesis mainly focuses on algorithmic and combinatorial questions related to some geometric problems on graphs. In the last part of this thesis, a graph coloring problem is also discussed.

Boxicity and Cubicity: These are graph parameters dealing with geometric representations of graphs in higher dimensions. Both these parameters are known to be NP-Hard to compute in general and are even hard to approximate within an $O(n^{1-\epsilon})$ factor for any $\epsilon > 0$, under standard complexity theoretic assumptions.

We studied algorithmic questions for these problems, for certain graph classes, to yield efficient algorithms or approximations. Our results include a polynomial time constant factor approximation algorithm for computing the cubicity of trees and a polynomial time constant (≤ 2.5) factor approximation algorithm for computing the boxicity of circular arc graphs. As far as we know, there were no constant factor approximation algorithms known previously, for computing boxicity or cubicity of any well known graph class for which the respective parameter value is unbounded.

We also obtained parameterized approximation algorithms for boxicity with various edit distance parameters. An $o(n)$ factor approximation algorithm for computing the boxicity and cubicity of general graphs also evolved as an interesting corollary of one of these parameterized algorithms. This seems to be the first sub-linear factor approximation algorithm known for computing the boxicity and cubicity of general graphs.

Planar grid-drawings of outerplanar graphs: A graph is outerplanar, if it has a planar embedding with all its vertices lying on the outer face. We give an efficient algorithm to 2-vertex-connect any connected outerplanar graph G by adding more edges to it, in order to obtain a supergraph of G such that the resultant graph is still outerplanar and its pathwidth is within a constant times the pathwidth of G. This algorithm leads to a constant factor approximation algorithm for computing minimum height planar straight line grid-drawings of outerplanar graphs, extending the existing algorithm known for 2-vertex connected outerplanar graphs.

Maximum matchings in triangle distance Delaunay graphs: Delaunay graphs of point sets are well studied in Computational Geometry. Instead of the Euclidean metric, if the Delaunay graph is defined with respect to the convex distance function defined by an equilateral triangle, it is called a Triangle Distance Delaunay graph. TD-Delaunay graphs are known to be equivalent to geometric spanners called half-Θ_6 graphs.

It is known that classical Delaunay graphs of point sets always contain a near perfect matching, for non-degenerate point sets. We show that Triangle Distance Delaunay graphs of a set of n points in general position will always contain a matching of size $\left\lceil \frac{n-1}{3} \right\rceil$ and this bound is tight. We also show that Θ_6 graphs, a class of supergraphs of half-Θ_6 graphs, can have at most $5n - 11$ edges, for point sets in general position.

Heterochromatic Paths in Edge Colored Graphs: Conditions on the coloring to guarantee the existence of long heterochromatic paths in edge colored graphs is a well explored problem in literature. The objective here is to obtain a good lower bound for $\lambda(G)$ - the length of a maximum heterochromatic path in an edge-colored graph G, in terms of $\vartheta(G)$ - the minimum color degree of G under the given coloring. There are graph families for which $\lambda(G) = \vartheta(G) - 1$ under certain colorings, and it is conjectured that $\vartheta(G) - 1$ is a tight lower bound for $\lambda(G)$.

We show that if G has girth is at least $4 \log_2(\vartheta(G)) + 2$, then $\lambda(G) \geq \vartheta(G) - 2$. It is also proved that a weaker requirement that G just does not contain four-cycles is enough to guarantee that $\lambda(G)$ is at least $\vartheta(G) - o(\vartheta(G))$. Other special cases considered include lower bounds for $\lambda(G)$ in edge colored bipartite graphs, triangle-free graphs and graphs without heterochromatic triangles.