Rainbow Colouring and some Dimensional Problems in Graph Theory

(Abstract of PhD Thesis)

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This thesis touches three different topics in graph theory, namely, *rainbow colouring*, *product dimension* and *boxicity*.

Rainbow colouring An edge colouring of a graph is called a *rainbow colouring*, if every pair of vertices is connected by at least one path in which no two edges are coloured the same. The *rainbow connection number* of a graph is the minimum number of colours required to rainbow colour it. In this thesis we give upper bounds on rainbow connection number based on graph invariants like minimum degree, vertex connectivity, and radius. We also give some computational complexity results for special graph classes.

Product dimension The *product dimension* or *Prague dimension* of a graph G is the smallest natural number k such that G is an induced subgraph of a *direct product* of k complete graphs. In this thesis, we give upper bounds on the product dimension for forests, bounded treewidth graphs and graphs of bounded degeneracy.

Boxicity and cubicity The *boxicity* (*cubicity*) of a graph G is the smallest natural number k such that G can be represented as an intersection graph of axis-parallel rectangular boxes (axis-parallel unit cubes) in $\mathbb{R}^k$. In this thesis, we study the boxicity and the cubicity of *Cartesian*, *strong* and *direct products* of graphs and give estimates on the boxicity and the cubicity of a product graph based on invariants of the component graphs.

Separation dimension The *separation dimension* of a hypergraph H is the smallest natural number k for which the vertices of H can be embedded in $\mathbb{R}^k$ such that any two disjoint edges of H can be separated by a hyperplane normal to one of the axes. While studying the boxicity of line graphs, we noticed that a box representation of the line graph of a hypergraph has a nice geometric interpretation. Hence we introduced this new parameter and did an extensive study of the same.