Abstract

The classical graph edge coloring problem deals in coloring the edges of a given graph with minimum number of colors such that no two adjacent edges in the graph, get the same color in the proposed coloring. In the following work, we look at the other end of the spectrum where in our goal is to maximise the number of colors used for coloring the edges of the graph under some vertex specific constraints.

We deal with the MAXIMUM EDGE COLORING problem which is defined as the following-For an integer $q \geq 2$ and a graph G, the goal is to find a coloring of the edges of G with the maximum number of colors such that every vertex of the graph sees at most q colors. The question is very well motivated by the problem of channel assignment in wireless networks. This problem is NP-hard for $q \geq 2$, and has been well-studied from the point of view of approximation.

This problem has not been studied in the parameterized context before. Hence as a next step, this thesis investigates the *parameterized complexity* of this problem where the standard parameter is the solution size. The main focus of the work is the special case of $q = 2$, i.e. MAXIMUM EDGE 2-COLORING which is theoretically intricate and practically relevant in the wireless networks setting.

We first show an exponential kernel for the MAXIMUM EDGE q-COLORING problem where q is a fixed constant and $q \geq 2$. We do a more specific analysis for the kernel of the MAXI-

MUM EDGE 2-COLORING problem. The kernel obtained here is still exponential in size but is better than the kernel obtained for MAXIMUM EDGE q-COLORING problem in case of $q = 2$.

We then show a fixed parameter tractable algorithm for the MAXIMUM EDGE 2-COLORING problem with a running time of $O^*(k^{O(k)})$. We also show a fixed parameter tractable algorithm for the MAXIMUM EDGE q-COLORING problem with a running time of $O^*(k^{O(qk)} \cdot q^{O(k)})$.

The fixed parameter tractability of the *dual parametrization* of the MAXIMUM EDGE 2-COLORING problem is established by arguing a linear vertex kernel for the problem. We also show that the MAXIMUM EDGE 2-COLORING problem remains *hard* on graphs where the maximum degree is a constant and also on graphs without cycles of length four. In both these cases, we obtain quadratic kernels.

A closely related variant of the problem is the question of MAX EDGE $\{1,2\}$-COLORING. For this problem, the vertices in the input graph may have different $q \in \{1,2\}$ values and the goal is to use at least k colors for the edge coloring of the graph such that every vertex sees at most q colors, where q is either one or two. We show that that the MAX EDGE $\{1,2\}$-COLORING problem is W[1]-hard on graphs that have no cycles of length four.