Abstract

Rotating and non-rotating beams are widely used to model important engineering structures. Hence, the vibration analyses of these beams are an important problem from a structural dynamics point of view. Depending on the beam dimensions, they are modeled using different beam theories. In most cases, the governing differential equations of these types of beams do not yield any simple closed-form solutions; hence we look for the inverse problem approach in determining the beam property variations given certain solutions.

The long and slender beams are generally modeled using the Euler-Bernoulli beam theory. Under the premise of this theory, we study (i) the second mode tailoring of non-rotating beams having six different boundary conditions, (ii) closed-form solutions for free vibration analysis of free-free beams, (iii) closed-form solutions for free vibration analysis for gravity-loaded cantilever beams, (iv) closed-form solutions for free vibration analysis of rotating cantilever and pinned-free beams and (v) beams with shared eigenpair. Short and thick beams are generally modeled using the Timoshenko beam theory. Here, we provide analytical closed-form solutions for the free vibration analysis of rotating non-homogeneous Timoshenko beams. The Rayleigh beam provides a marginal improvement over the Euler-Bernoulli beam theory without venturing into the mathematical complexities of the Timoshenko beam theory. Under this theory, we provide closed-form solutions for the free vibration analysis of cantilever Rayleigh beams under three different axial loading conditions - uniform loading, gravity-loading and centrifugally loaded.

We assume simple polynomial mode shapes which satisfy the different boundary conditions of a particular beam, and derive the corresponding beam property variations. In case of the shared eigenpair, we use the mode shape of a uniform beam which has *a closed-form solution and use it to derive the stiffness distribution of a corresponding axially loaded beam having same length, mass variation and boundary condition. For the Timoshenko beam, we assume polynomial functions for the bending displacement and the rotation due to bending. The derived properties are demonstrated as benchmark analytical solutions for approximate and numerical methods used for the free vibration analysis of beams. They can also aid in designing actual beams for a pre-specified frequency or nodal locations in some cases. The effect of different parameters in the derived property variations and the bounds on the pre-specified frequencies and nodal locations are also studied for certain cases.*

The derived analytical solutions can also serve as a benchmark solution for different statistical simulation tools to find the probabilistic nature of the derived stiffness distribution for known probability distributions of the pre-specified frequencies. In presence of uncertainty, this flexural stiffness is treated as a spatial random field. For known probability distributions of the natural frequencies, the corresponding distribution of this field is determined analytically for the rotating cantilever Euler-Bernoulli beams. The derived analytical solutions are also used to derive the coefficient of variation of the stiffness distribution, which is further used to optimize the beam profile to maximize the allowable tolerances during manufacturing.