Abstract

The finite element method (FEM) is a widely-used numerical tool in the fields of structural dynamics, acoustics and electromagnetics. In this work, our goal is to develop robust FEM strategies for solving problems in the areas of acoustics, structures and electromagnetics, and then extend these strategies to solve multi-physics problems such as magnetohydrodynamics and structural acoustics. We now briefly describe the finite element strategies developed in each of the above domains.

In the structural domain, we show that the trapezoidal rule, which is a special case of the Newmark family of algorithms, conserves linear and angular momenta and energy in the case of undamped linear elastodynamics problems, and an 'energy-like measure' in the case of undamped acoustic problems. These conservation properties, thus, provide a rational basis for using this algorithm. In linear elastodynamics variants of the trapezoidal rule that incorporate 'high-frequency' dissipation are often used, since the higher frequencies, which are not approximated properly by the standard displacement-based approach, often result in unphysical behavior. Instead of modifying the trapezoidal algorithm, we propose using a hybrid FEM framework for constructing the stiffness matrix. Hybrid finite elements, which are based on a two-field variational formulation involving displacement and stresses, are known to approximate the eigenvalues much more accurately than the standard displacement-based approach, thereby either bypassing or reducing the need for high-frequency dissipation. We show this by means of several examples, where we compare the numerical solutions obtained using the displacement-based and hybrid approaches against analytical solutions. We also present a monolithic formulation for the solution of structural acoustic problems based on the hybrid finite

element approach.

In the area of electromagnetics, since our goal is to ultimately couple the electromagnetic analysis with structural or fluid variables in a 'monolithic' framework, we focus on developing nodal finite elements rather than using 'edge elements'. It is well-known that conventional nodal finite elements can give rise to spurious solutions, and that they cannot capture singularities when the domains are nonconvex and have sharp corners. The commonly used remedies of either adding a penalty term or using a potential formulation are unable to address these problems satisfactorily. In order to overcome this problem, we first develop several *mixed* finite elements in two and three dimensions which predict the eigenfrequencies (including their multiplicities) accurately, even for non-convex domains. In this proposed formulation, no ad-hoc terms are added as in the penalty formulation, and the improvement is achieved purely by an appropriate choice of the finite element spaces for the different variables. For inhomogeneous domains, 'double noding' is used to enforce the appropriate continuity conditions at an interface. Although the developed mixed FEM works very accurately for all 2D geometries and regular Cartesian 3D geometries, it has so far not yielded success for curved 3D geometries. Therefore, for 3D harmonic and transient analysis problems, we propose and use a *modified* form of the potential formulation that overcomes the disadvantages of the standard potential method, especially on non-convex domains.

Electromagnetic radiation and scattering in an exterior domain traditionally involved imposing a suitable absorbing boundary condition (ABC) on the truncation boundary of the numerical domain to inhibit reflection from it. In this work, based on the Wilcox asymptotic expansion of the electric far-field, we propose an amplitude formulation within the framework of the nodal FEM, whereby the highly oscillatory radial part of the field is separated out a-priori so that the standard Lagrange interpolation functions have to capture a relatively gently varying function. Since these elements can be used in the immediate vicinity of the radiator or scatterer (with few exceptions which we enumerate), it is more effective compared to methods of imposing ABCs, especially for high-frequency problems. We show the effectiveness of the proposed formulation

on a wide variety of radiation and scattering problems involving both conducting and dielectric bodies, and involving both convex and non-convex domains with sharp corners.

The Time Domain Finite Element Method (TDFEM) has been used extensively to solve transient electromagnetic radiation and scattering problems. Although conservation of energy in electromagnetics is well-known, we show in this work that there are additional quantities that are also conserved in the absence of loading. We then show that the developed time-stepping strategy (which is closely related to the trapezoidal rule) mimics these continuum conservation properties either exactly or to a very good approximation. Thus, the developed numerical strategy can be said to be 'unconditionally stable' (from an energy perspective) allowing the use of arbitrarily large time-steps. We demonstrate the high accuracy and robustness of the developed method for solving both interior and exterior domain radiation problems, and for finding the scattered field from conducting and dielectric bodies.

In the field of magneto-hydrodynamics, we develop a monolithic strategy based on a continuous velocity-pressure formulation that is known to satisfy the Babuska-Brezzi (BB) conditions. The magnetic field is interpolated in the same way as the velocity field, and the entire formulation is within a nodal finite element framework. Both transient and steady-state formulations are developed for two- and three-dimensional geometries. An exact linearization of the monolithic strategy ensures that rapid (quadratic) convergence is achieved within each time (or load) step, while the stable nature of the interpolations used ensure that no instabilities arise in the solution. Good agreement with analytical solutions, even with the use of very coarse meshes, shows the efficacy of the developed formulation.