Abstract

The classical n-bugs problem has attracted considerable attention from researchers. This problem stems from the study of movement of a group of animals. In the context of multi-agent systems the problem has been modelled as cyclic pursuit. Under this paradigm, every agent, indexed i, chases its unique leader, agent $i + 1$ (modulo n), with n being the total number of agents.

In the existing literature, cyclic pursuit has been studied for homogeneous agents where each agent's velocity is proportional to the distance separating it from its leader and is directed along the line joining it to its leader. The constant of proportionality, initially chosen to be the same for all the agents, resulted in consensus in position, without the need for any centralized controller. Later, the constant of proportionality, alternately called the gain, was allowed to be heterogeneous and positional consensus was still achieved. Moreover, it was shown that the point of convergence, where the agents rendezvous, could be chosen at will, except for some diagnostic cases. In this thesis, besides admitting heterogeneous gains, the agents are assumed to pursue their respective leaders with an angle of deviation from the line joining them to their corresponding leaders. This expands the reachability set (set of points where the agents can rendezvous) for the system of agents to include points that were hitherto unreachable. Sufficient conditions for stability of such systems have been derived in this thesis. Detailed analysis of the reachability set has also been carried out.

Some researchers have also investigated hierarchical cyclic pursuit, where there are multiple levels of pursuit. For instance, in the two level hierarchical pursuit, the agents are divided into m groups of n agents each, where each agent in a group chases its leader within the group as well as a similarly indexed agent in its leading group. Thus, groups of agents are also in cyclic pursuit. So far, only homogeneous gains were considered under this paradigm. The present thesis admits heterogeneous gains and establishes necessary and sufficient conditions for the stability of heterogeneous hierarchical cyclic pursuit, that generalize existing results. Reachable sets are also derived for this case. It is proved that the existing results can be derived as special cases of the ones considered in this thesis.

As an extension to a realistic application, the importance of expansion in reachable set vis-à-vis capturing a moving target is highlighted in this thesis. It has been shown that if the target's initial position is reachable, then using a control law proposed in the thesis, the target can be captured. This control law is essentially an augmented cyclic pursuit law with the target's velocity information fed to each agent in addition to the conventional cyclic pursuit command. Analysis has been carried out for agents with double integrator dynamics as well. A control law in conjunction with an algorithm is proposed that helps ensure global reachability of agents, with double integrator dynamics, in cyclic pursuit.

Another application, in which cyclic pursuit and a closely related topology called platooning have been coupled together to track the boundaries of unknown regions and constantly monitor them, is addressed in this thesis. This problem is especially important in monitoring forest fire, marine contamination, volcanic ash eruptions, etc., and can protect human life by cordoning off unsafe regions using multiple autonomous agents.

Lastly, discrete time cyclic pursuit laws are analyzed to obtain results similar to the continuous time counterparts that exist in the literature. Moreover, heterogeneous gains and deviations are admitted similar to the continuous time version considered in this thesis. Gershgorin's theorem is used extensively to arrive at sufficient conditions for the stability of such discrete time deviated cyclic pursuit systems. Reachability sets are also derived. In case of discrete time systems, loss of synchronization due to no common clock for autonomous agents is a very realistic scenario. This thesis obtains some results on the stability of such asynchronous cyclic pursuit systems and indicates that special precautions are needed for dealing with heterogeneous cyclic pursuit systems even when one gain is negative, since the system may not converge, depending on the initial positions of the agents and the sequence of updates.