

# Preface

The subject of structural reliability provides a logical framework within which the uncertainties associated with problems of structural analysis, design, health monitoring and maintenance, can be systematically addressed. Here, the uncertainties in structural and load characteristics and structural modelling are quantified using the mathematical theories of probability, random variables, random processes and statistics. The subject essentially aims to establish relationship between probability of structural failure and the uncertainty in parameters connected with the structural and load characteristics and structural modelling. This, in turn, facilitates a rational basis for deciding upon optimal structural configurations for a given set of loading conditions, consistent with desired levels of safety and affordable cost. In fact, many of the present day codes of practice for structural design employ concepts of limit state and partial load and resistance factors and have been calibrated based on probabilistic modelling of uncertainties. Failure of important structures, such as dams, nuclear plants and other industrial installations, leads to large scale loss of life and property. These structures need to be designed to withstand rare events, such as earthquakes of large magnitudes, with specified levels of probability, consistent with acceptable risk levels in the society. Reliability estimation of such important structures is, thus, of primary concern in structural engineering. This concern has been a driving force in the development of methods and tools for the assessment of structural reliability.

The focus of the present study has been on research into methods for prediction of reliability of randomly parametered dynamic structures, subjected to random vibratory inputs. Specific themes explored in this thesis include development of multivariate extreme value distributions and their applications in time variant reliability analysis of structural systems, application of methods of structural reliability analysis in seis-

mic fragility analysis, extreme value distribution of non-Gaussian random processes, such as Von Mises' stress metric, development of response surface methods for time variant reliability analysis involving performance functions with multiple design points and/or multiple regions of comparable importance and issues related to discretization of non-Gaussian random fields in stochastic finite element analysis of random dynamic structures. The thesis is organized into seven chapters and three appendices.

The first chapter contains an overview of the subject of structural reliability and the methods available for computing failure probabilities. The topics covered include probabilistic modelling of uncertainty (including discussions on non-Gaussian models), first and second order reliability methods, asymptotic analysis of the probability integral, simulation based techniques (including variance reduction methods), system reliability analysis, time variant reliability analysis, probabilistic model reduction and importance measures, stochastic finite element method (including discussions on simulation and discretization of Gaussian/non-Gaussian random fields) and response surface methods. The review identifies a set of open problems that are subsequently addressed in the thesis.

Chapter 2 focusses on the development of the joint probability distribution of extreme values associated with a multi-dimensional vector of mutually correlated, stationary, Gaussian random processes. The multivariate extreme value distribution functions have applications in reliability analysis of structural systems, characterized by multiple limit states. A solution to this problem is developed by modelling the counting process, associated with the number of level crossings of a set of mutually correlated, stationary, Gaussian random processes, as a multivariate Poisson random process. This, in turn, leads to approximations to the multivariate probability distributions for the first passage times and extreme values, over a given duration. It is shown that the multivariate extreme value distribution has Gumbel marginals and the first passage time has exponential marginals. The acceptability of the solutions developed is examined by performing simulation studies on bivariate Gaussian random processes. Illustrative examples include a discussion on the response analysis of a two span bridge, subjected to spatially varying random earthquake support motions.

The formulations developed in Chapter 2, are extended in Chapter 3, to include structure randomness in the reliability calculations. Attention is focussed on linear structure behavior. The structure randomness is modeled through a vector of mutually correlated,

non-Gaussian random variables. The excitation components are assumed to be jointly stationary, Gaussian random processes. The structure is defined to be safe, if a set of response quantities, in their steady states, remains within prescribed thresholds, over a given duration of time. Estimates of the failure probability, conditioned on structure randomness, are first obtained using the theory of multivariate extremes developed in Chapter 2. Subsequently, a method based on Taylor's series expansion, is developed, for estimating the unconditional failure probability. This involves the evaluation of gradients of the multivariate extreme value distribution, conditioned on structure random variables, with respect to the basic variables. Discussions on analytical and numerical techniques for computing these gradients are presented. The proposed method is illustrated through two numerical examples.

The problem of determining the probability distribution function of extremes of Von Mises stress, over a specified duration, in linear vibrating structures subjected to stationary, Gaussian random excitations, is considered in Chapter 4. In the steady state, the Von Mises stress is a stationary, non-Gaussian random process, with non-zero mean. The number of times the process crosses a specified threshold in a given duration is modeled as a Poisson random variable. The determination of the parameter of this model, in turn, requires the knowledge of the joint probability density function of the Von Mises stress and its time derivative. Alternative models for this joint probability density function, based on translation process model, combined Laguerre-Hermite polynomial expansion and the maximum entropy model are considered. In implementing the maximum entropy method, the unknown parameters of the model are derived by solving a set of linear algebraic equations, in terms of the marginal and joint moments of the process and its time derivative. At any given time instant, the Von Mises stress and its time derivative are uncorrelated but nevertheless are mutually dependent. This feature is essentially reflective of the non-Gaussian nature of the Von Mises stress process. The proposed method is shown to be capable of taking into account this feature. For the purpose of illustration, the extremes of the Von Mises stress in a pipe support structure under random earthquake loads, are examined. The results on extremes of the Von Mises stress, based on maximum entropy model, are shown to compare well with Monte Carlo simulation results.

A response surface based procedure is developed in Chapter 5 for estimating the time variant reliability of randomly driven linear/nonlinear vibrating structures. Here, it

is assumed that the performance functions are defined implicitly through a computer code, such as, a finite element analysis program. The focus of the study is on developing response surface models for performance functions that possess multiple design points and/or multiple regions of comparable importance. The excitations are considered to be non-stationary Gaussian processes. The structure properties are modeled as non-Gaussian random variables. The structural responses are therefore, non-Gaussian processes, the distributions of which are not generally available in an explicit form. The limit state is formulated in terms of the extreme value distribution of the response random process. The basic feature of the proposed method involves fitting a global response surface which approximates the limit surface in regions which make significant contributions to the failure probability. Failure probability estimates are, subsequently, obtained by carrying out Monte Carlo simulations on the fitted response surface. The method is integrated with commercial finite element softwares which enable reliability analysis of large structures with complexities that include material and geometric nonlinear behavior. A suite of illustrative examples are presented to demonstrate the capabilities of the method.

The developments in Chapters 2-5 assume random variable representation for the structure parameter uncertainties. Questions related to the application of these methods, when random field models are adopted for representing spatial structure inhomogeneities, are addressed in Chapter 6. Stochastic finite element based formalisms are developed to address these issues. With this in view, the problem of characterizing response variability and reliability analysis of skeletal vibrating structures, made up of randomly inhomogeneous, curved/straight Timoshenko beams, is considered. The excitation is taken to be random in nature. A frequency domain stochastic finite element method is developed in terms of dynamic stiffness coefficients of the constituent stochastic beam elements. The displacement fields are discretized by using frequency and damping dependent shape functions. Questions related to discretizing the inherently non-Gaussian random fields that characterize beam elastic, mass and damping properties are considered. Analytical methods, combined analytical and simulation based methods, direct Monte Carlo simulations and simulation procedures that employ importance sampling strategies are brought to bear on analyzing dynamic response variability and assessment of structure reliability. Satisfactory performance of approximate solution procedures outlined in the study is demonstrated using limited Monte Carlo simulations.

A brief summary of contributions made in the thesis together with a few suggestions for further research are presented in Chapter 7

Appendix A presents a case study in seismic fragility analysis of a piping structure of a nuclear power plant. The framework for the fragility calculations incorporate the developments in Chapters 2 and 3. The seismic excitations are modeled as a vector of random processes and detailed models of the structures are constructed using finite elements. The fragility calculations carried out, here, encompass the finite element method for detailed structure analysis, random vibration theories for characterizing structure response variabilities and reliability based methods for estimating structure failure probabilities.

The framework for the fragility calculations in Appendix A require that codes on reliability based methods and random vibration analysis need to be executed in conjunction with commercially available finite element softwares. Sample programs developed for interfacing a commercially available finite element software and MATLAB codes which employ reliability based methods are presented in Appendix B. Appendix C provides details of sample MATLAB codes used for computing joint moments of Von Mises stress and its instantaneous time derivative. These codes constitute an important part in the development of the method proposed in Chapter 4.