

Synopsis

It is known that high-speed flows are compressible. In large parts of the flow domains, the inviscid approximation is valid and this leads to Euler equations of gas dynamics. These inviscid compressible flows are modelled by coupled nonlinear hyperbolic systems of partial differential equations and generally require numerical solution techniques, as analytical solutions are usually not available.

Out of all the numerical methods developed over the past five decades to solve the Euler equations, the schemes based on kinetic theory of gases are elegant ones with distinct advantages of simplicity and robustness. However, many kinetic or Boltzmann schemes suffer from high dose of numerical diffusion and these methods are known to be less accurate. The exact shock capturing of steady grid-aligned discontinuities, achieved at the macroscopic level, is yet to be claimed by this class of methods. A closely related class of discrete velocity Boltzmann schemes proved to be advantageous in this regard, with the first discrete kinetic scheme with exact shock capturing being introduced by Raghurama Rao and Balakrishna[51], by enforcing the Rankine-Hugoniot jump condition at the discrete level. In the first part of this thesis, this accurate shock capturing algorithm with a relaxation system is further improved by various techniques, such as including a diagonal matrix of coefficient of numerical diffusion for vector cases, introducing a wave speed correction mechanism for obtaining physically realistic solutions, introducing a limiter based variant to avoid the use of an entropy χ and finally modifying the numerical diffusion based on the entropy conservation equation to obtain a simple entropy stable and yet accurate discrete velocity Boltzmann scheme. The features of all the new variants are demonstrated by application to several bench-mark test problems.

In the second part of the thesis, a discrete velocity Boltzmann scheme which can capture steady contact discontinuities exactly is developed by using the generalized Riemann invariants together with the jump conditions. This scheme is accurate and widely applicable, without the need for any entropy correction, the relevant features being demonstrated by application to several benchmark test problems.

In the third part of this thesis, a discrete velocity Boltzmann scheme is developed by using physically relevant discrete velocities. A derivation introduced by Sanders and Prendergast [58]

is modified to introduce the velocities, of the Dirac delta functions which replace the Maxwellian, matching the eigenvalues at the macroscopic level. This strategy is further coupled with the framework of a discrete velocity Boltzmann system to develop an efficient relaxation scheme for solving the Euler equations. This new algorithm is found to be low in numerical diffusion and also successful in handling various challenging test problems.