

## Abstract

Solution of Maxwell's equations is required to model and analyse electromagnetic behaviour of circuits, components and systems. These equations being partial differential equations of vector field quantities with spatial and time variations, are difficult to solve analytically except for a handful of problems. Numerical approaches such as finite difference time domain (FDTD) method, finite element method (FEM) and method of moments (MOM) etc. have been developed and a several commercial software packages are available based on these methods. However, these commercial tools based on FEM struggle to analyse complex geometries especially as their size becomes large in wavelengths. One example is the analysis of cold-test parameters of millimetre wave devices such as a travelling wave tube (TWT) slow wave structures (SWS). This research work aims at developing core capability to address such computational problems by formulating and implementing a finite element code and validating this in closed domain electromagnetic (EM) problems.

Due to the complex nature of the FEM implementation, a simpler 1D formulation by restructuring transfer matrix for layered EM problems has been attempted first. This approach is often known as spectral element method in structural analysis. We have demonstrated the utility of this approach for analysis and synthesis of simple layered EM structures such as radar absorbing materials. Limitations of this essentially scalar field formulation are addressed by implementing nodal and edge finite element-based formulations in this thesis.

This thesis further extends core capability of our in-house 2D and 3D finite element software's for EM analysis of periodic structures such as a TWT. Towards this, the base finite element solver for the wave propagation analysis of a 2D EBG structure using Floquet Bloch modes has been implemented first. Considering that the permittivity of the composite dielectric EBG structures may have random variations, a spectral decomposition technique, namely the Korhonen-Lotief Expansion, has been formulated for use in conjunction with the finite element method to compute the statistical behaviour of the response. More complex periodic cells with multiple domain regions embedded in a host matrix has also been analysed by making use of the domain independency of the KL expansion. The results obtained with the proposed approach has been validated using conventional statistical approaches such as Monte Carlo method. However, the proposed approach is significantly faster.

Only by extending the finite element method to a full 3D FEM solver more complex geometries such as slow wave TWT structures can be analysed accurately. Therefore, the 3D FEM code with complete post processing procedures for computing the cold test parameters has been developed. Paoletti mapping is used in 3D FEM core algorithm to efficiently handle edge elements for fast computation of EM problems with complex geometries. The basic dispersion diagram has been validated with commercial software. Comparison show significant advantage in the computation time. As part of post-processing capabilities developed, in addition to the extended dispersion characteristics derived from eigen mode analysis, the on-axis interaction impedance (Pierce impedance) has been computed by implementing ray tetrahedron intersection technique on the FEM mesh. These implementations are validated by modelling geometries from the literature.

Furthermore, an extension has been developed to handle finite conducting walls. Considering the limitations of various available fabrication technologies, the accurate model of surface roughness of the lossy walls has been considered in the simulation analysis. The surface impedance has been modelled as a random field to better account for the spatial distribution of surface roughness. The stochastic analysis has been performed by KL discretization of the impedance matrix. The statistics of the unit cell gain, and phase velocity has been estimated and validated against Monte Carlo simulations.

Analysis of SWS has been further extended to incorporate tolerance in geometric dimensions. The dimensional variations of the structure have been accounted for by scaling the Jacobian of the affine geometric parameter in the corresponding finite element integrals. This gives a stochastic matrix eigenvalue equation. To solve the random eigenvalue problem, stochastic collocation technique using Hermite quadrature points with proper response surface interpolation polynomials have been developed. The influence of the random variations of the geometric parameters on the critical response parameters have been quantified. These stochastic collocation results are compared with Monte Carlo simulations. In the case of Monte Carlo analysis of geometric parameter variations, a new finite element mesh has been generated for each random sample by modifying the .GEO file of GMSH using MATLAB. On the other hand, a single base mesh can be used in the proposed spectral collocation approach for quantifying geometric uncertainties which makes it an efficient and fast method for such analyses.

In summary, this thesis presents the development of core in-house capability with the implementation of finite element tool for EM structures using both nodal and edge finite element formulations. After validations, additional capabilities have been implemented for statistical variations in the material properties of the model space and dimensions of select elements of the model.