Thesis Title: ***Constrained Optimal Guidance Design of Interceptors for High-Speed Ballistic Targets***

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ABSTRACT

In this thesis, optimal control theory has been applied to design constrained terminal guidance algorithms for interceptor to destroy incoming high speed ballistic targets. There exist several optimal guidance algorithms which are developed to achieve different mission objectives. Two types of terminal angle-constrained optimal guidance laws are studied here. The first one has a closed form expression which is obtained by minimizing a weighted control energy considering linear state dynamics with states as zero-effort miss (ZEM) and zero-effort velocity miss (ZEVM). This guidance law is very popular due to its simple geometric interpretation and easy mechanization and is known as 'Generalized Vector Explicit Guidance' (GENEX). The second one has a closed from control update expression obtained by minimizing control energy considering nonlinear dynamic model of the interceptor. This guidance law is predictive in nature, converges in little iterations, and convergence time is very small. It is very accurate in achieving the terminal constraints. It is known as 'Model Predictive Static Programming' (MPSP).

In GENEX formulation, a fixed final time (i.e. time-to-go) optimal control problem has been solved to obtain a closed form guidance law. It is found that, GENEX guidance depends on time-to-go and final velocity. Selection of a different final time the problem becomes different, which leads to a different solution. Hence, there exists an optimal final time for which the cost function will become minimum among all control effort minimizing solutions, leading to the minimum control effort solution. In this thesis, the scope of finding optimal final time is explored and a closed form expression of time-to-go is obtained, which finds minimum of all minimum cost. In addition to it, the magnitude of the final velocity vector is obtained by deriving an analytic expression using nonlinear dynamics of the interceptor. It has been found that use of optimal time-to-go and value of final velocity has reduced the cost, which makes GENEX more efficient and cost effective.

MPSP is a numerical optimization algorithm which solves an optimal control problem with terminal constraints. The dimension of optimization problem is same as the number of discrete time instants. In this thesis, an attempt has been made to reformulate MPSP to reduce the number of optimization variables by representing the control variable as weighted sum of basis functions. This variety of MPSP is named as 'Quasi Spectral MPSP' (QS-MPSP). It has been found that, QS-MPSP formulation greatly reduces dimension of the optimization problem and consequently convergence time of the algorithm also reduced.

A successful interception requires the target to be confined all the time within the field of view of the seeker antenna. For this reason, the 'Look Angle' (i.e. the angle between the body '*x*' axis and Line of Sight) needs to be lesser than field of view angle of seeker antenna. If the angle of attack and side-slip are assumed to be small then look angle can be approximated as lead angle (defined as the angle between the velocity and LOS vector). Therefore, the lead angle should be lesser than the field of view angle during the terminal phase of an engagement. Hence, it can be considered as a path constraint optimal control problem. There exist direct and indirect methods to solve constrained optimal control problems. In this thesis the constrained optimal control problem is solved by a new technique which is developed in QS-MPSP frame work and is named as 'Constrained QS-MPSP. The idea behind this technique is to express the constraints on states in terms of inequality constraints on coefficients which are used to represent the control expression in spectral form. Constraints on control is taken care of by deriving separate set of inequality constraints on the same coefficients. Values of these coefficients are obtained by solving a static optimization problem in which a quadratic function of these coefficients are minimized subject to a set of inequality constraints on coefficients. It can be mentioned that; the static optimization problem is solved using interior point method which is a well-established algorithm and capable of solving large dimension nonlinear programming problem with inequality constraints. The proposed technique is applied to missile guidance problem. It is found that, the algorithm converges in a few iterations and convergence time is very small. For this reason, this algorithm can be implemented in real time on an on-board missile embedded processor.

The guidance commands (lateral accelerations) generated by proposed Constrained QS-MPSP, are validated using Six degree-of-freedom model of interceptor. The guidance commands are converted to control surface defections using a two loop Dynamic Inversion controller. It has been found that, the engagement happened, and lead angle constraint is satisfied.