Abstract

This thesis deals with motion planning of flexible one-dimensional objects and hyper-redundant serial robots moving in a plane or in three dimensional space. The flexible one-dimensional object is modeled as a continuous curve and a point on the curve is given a prescribed displacement. The key problem studied in the thesis is to obtain the motion of all points on the curve for the prescribed displacement subject to the condition of the length of the curve being preserved. Such motions are motivated by the need to model, analyze and realistically render of motion of hair, ropes and, more recently, flexible endoscopes where the assumption of constant axial length is realistic and reasonable. In this thesis, the discretized form of the flexible one-dimensional object is related to hyper-redundant robots and motion planning for such robots are obtained when the robot moves in free space and in a cluttered environment, avoiding obstacles.

The motion planning of flexible one-dimensional objects is posed as an optimization problem with constraints and calculus of variation is employed to derive general analytical results. The first analytical result is that, for a given motion of a point on the curve and subject to the preservation of the length of the curve, the infinitesimal motion of any other point on the curve is minimized when the velocity vector at that point of the curve is along the tangent to the curve at that point. This leads to the second key result that when one end of a straight line segment is moved along a straight line, the velocity of the distal (far) end is minimized when it is along the straight line segment and the curve traced by the distal end is the well-known tractrix curve whose closed-form analytic expressions can be obtained using hyperbolic functions. If the flexible one-dimensional object is discretized by several piece-wise straight line segments, the magnitude of the velocity vector of the distal end of the segments attenuates as one goes away from the end where the input displacement is provided and if the direction of the input displacement is not changed, all the line segments eventually line up along the direction of the input displacement. It is shown that the attenuating and eventual aligning features lead to realistic and a more natural motion of the discretized segments and results in the establishment of a $\mathcal{O}(n)$ algorithm for motion planning. It is shown that the developed algorithm can be used for real-time simulation of the motion of discretized flexible one-dimensional objects and hyper-redundant serial robots.

For realistic simulation and rendering of the motion, the flexible object must be discretized into a large number of straight segments. In the second part of the thesis, the flexible one-dimensional object is represented by a spline and motion planning algorithm is applied to the segments of the underlying control polygon of the spline. Since the number of segments in a control polygon can be significantly less, a significant increase in efficiency in simulation and rendering of the motion is obtained. However, it is known that as the control polygon is moved, the length of the spline curve changes. To overcome this problem, an innovative adaptive algorithm, involving sub-division and merging of the segments of the control polygon, is presented and this restricts the variation in the length of the curve to within a user prescribed tolerance. New analytical results related to the length of the curve and the angle between the adjacent segments of the control polygon are derived for quadratic and cubic splines and, depending on the prescribed tolerance, threshold values of the angle are obtained and used in the algorithm for approximate length preservation.

The last part of the thesis deals with development of a planar hyper-redundant robot and implementation of motion planning algorithm on this robot. The hyper-redundant robot contains 12 links connected by actuated rotary joints which can change the angle between the links in a controlled manner. The links are on the wheels which provide support and allow it to move forward. The leading link also has a DC motor which can rotate the wheels so that it can move forward and pull the trailing links. Using the motion planning algorithm, for a prescribed motion of the leading link, the angle between two successive links are computed. These are given as input to the robot and the path traced by the 12 link robot is observed. It is seen that the motion of the hyper-redundant robot has the expected natural and realistic motion characteristics. It is furthermore demonstrated that the calculus of variation based approach for motion planning can be extended to include obstacle avoidance by adding additional constraints related to the location and size of the obstacles. It is shown that the entire robot optimally avoids the obstacles and moves in a more natural and realistic way.